



Factor trees

Composite numbers can be divided exactly (with no remainder), by other smaller or equal whole numbers called **factors**.

Composite numbers:	15	9	12	4	24
Factors:	1, 3, 5, 15	1, 3, 9	1, 2, 3, 4, 6, 12	1, 2, 4	1, 2, 3, 4, 6, 8, 12, 24

Prime numbers only have 1 and themselves as factors.

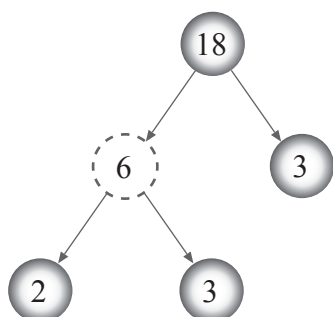
Prime numbers:	2	3	17	11	31
Factors:	1, 2	1, 3	1, 17	1, 11	1, 31

All composite numbers can be written as the product (\times) of **prime factors** (all the prime numbers that divide exactly into them). Let's see how.



'Express' is another way of saying 'write' in Mathematics.

Express 18 as a product of its prime factors



Split 18 into two smaller factors

Solid circle around prime numbers to stop that branch

Split 6 into two smaller factors

Solid circle around prime numbers to stop that branch

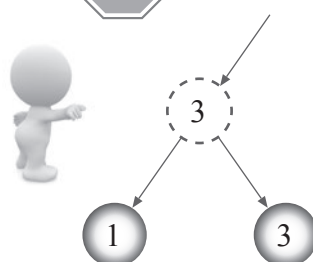
Once every branch has reached a prime number, multiply all the prime numbers together

$$\begin{aligned}\therefore 18 &= 2 \times 3 \times 3 \\ &= 2 \times 3^2\end{aligned}$$

Simplify answer

ALWAYS  at the prime number.

Don't ever do this



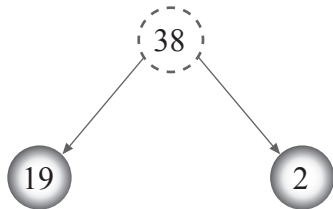
because 1 is NOT a prime number



Remember:
A prime number has two factors, itself and 1

Here are some more examples.

Express 38 as a product of its prime factors



Split 38 into two smaller factors

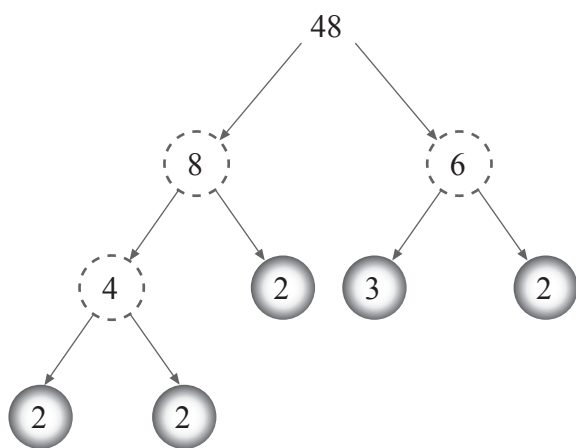
Solid circle around prime numbers to stop that branch

Once every branch has reached a prime number, multiply all the prime numbers together

$$\therefore 38 = 19 \times 2$$

There is often more than one way to create a factor tree for numbers with a lot of factors.

Express 48 as a product of its prime factors



Split 48 into two smaller factors

Split 6 and 8 into two smaller factors

Solid circle around prime numbers to stop that branch

Split 4 into two smaller factors

Solid circle around prime numbers to stop that branch

Once every branch has reached a prime number, multiply all the prime numbers together

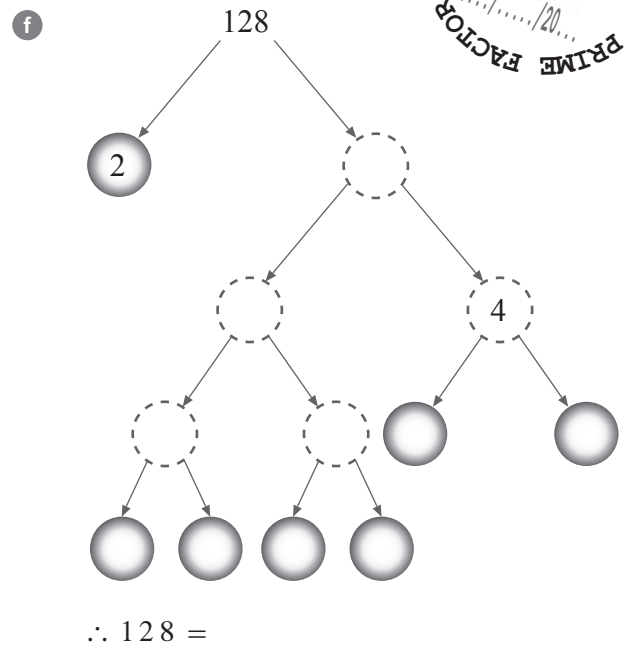
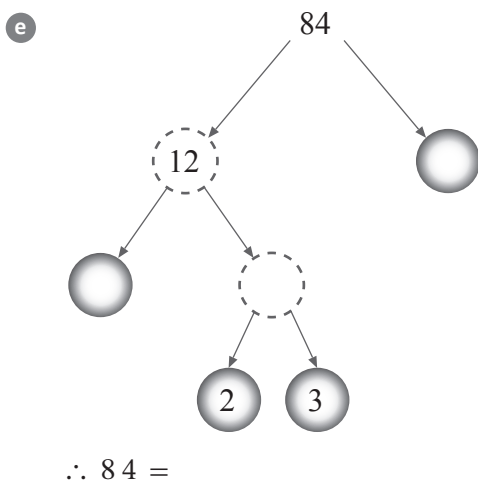
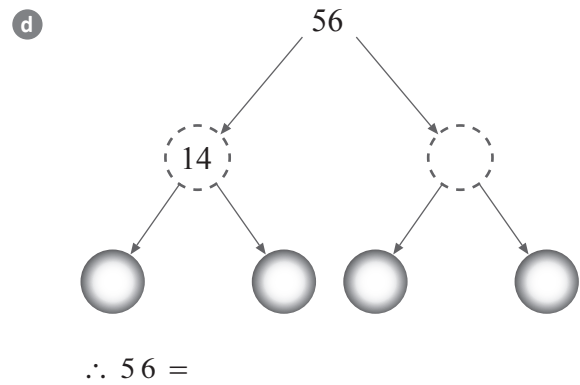
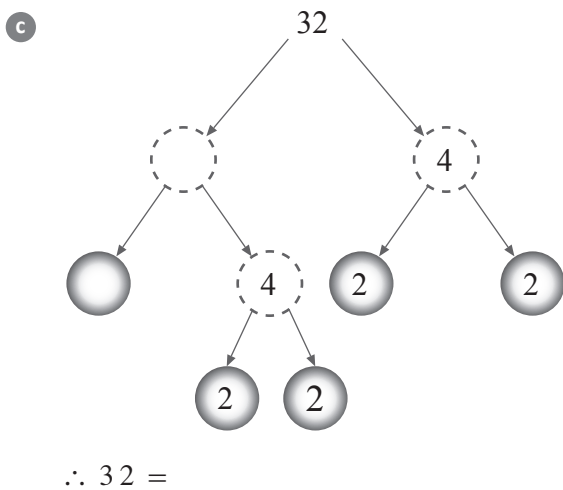
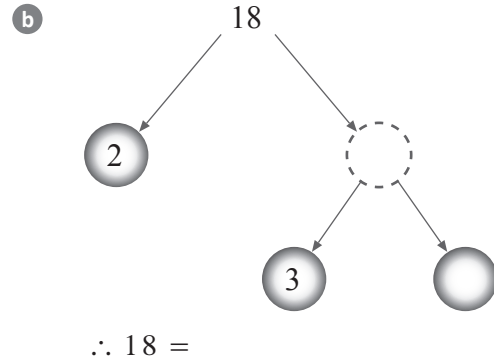
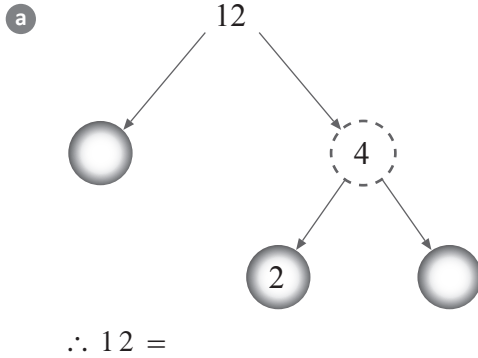
$$\begin{aligned}\therefore 48 &= 2 \times 2 \times 2 \times 3 \times 2 \\ &= 2^4 \times 3\end{aligned}$$

Simplify answer



Factor trees

1 Fill in the missing values on the following factor trees and write the number as a product of its primes.



**Factor trees**

2 Complete a factor tree for each number below and express them as a product of their prime factors.

a

8

b

20

 $\therefore 8 =$ $\therefore 20 =$

c

24

d

60

 $\therefore 24 =$ $\therefore 60 =$

e

96

f

144

 $\therefore 96 =$ $\therefore 144 =$

Highest common factor (HCF)

The HCF is the largest number that divides exactly into two or more composite numbers.

Write all the factors of each number then circle the largest one which appears in both lists.

Find the highest common factor for these pairs of numbers

(i) 6 and 8

Factors of 6: 1, 2, 3, 6

List all the factors for each number

Factors of 8: 1, 2, 4, 8

Circle the largest number common to both lists

∴ The HCF for 6 and 8 is: 2

(ii) 18 and 12

Factors of 18: 1, 2, 3, 6, 9, 18

List all the factors for each number

Factors of 12: 1, 2, 3, 4, 6, 12

Circle the largest number common to both lists

∴ The HCF for 18 and 12 is: 6

We can use the list of prime factors for larger numbers to find the HCF.

Find the HCF for these pairs of larger numbers

(i) 72 and 96

Factors of 72: 2, 2, 2, 3, 3

List all the **prime** factors for each number

Factors of 96: 2, 2, 2, 2, 2, 3

∴ The HCF for 72 and 96 is: $2 \times 2 \times 2 \times 3 = 24$

(ii) 528 and 624

Factors of 528: 2, 2, 2, 2, 3, 11

List all the **prime** factors for each number

Factors of 624: 2, 2, 2, 2, 3, 13

∴ The HCF for 528 and 624 is: $2 \times 2 \times 2 \times 2 \times 3 = 48$

**Highest common factor (HCF)**

1 Find the highest common factor for these pairs of numbers.

a 8 and 12

b 6 and 15

c 10 and 18

d 18 and 24

e 14 and 28

f 16 and 36

2 Use the prime factors to find the HCF for these larger numbers.

a 42 and 84

b 92 and 72

c 280 and 490

d 256 and 640



Lowest common multiple (LCM)

The LCM is the smallest number that is common to the multiplication tables of two or more numbers.

Write down the multiples of the numbers and stop once you find the lowest common multiple.

Find the lowest common multiple for these pairs of numbers

(i) 2 and 5

Multiples of 2: 2×2 , 4×2 , 6×2 , 8×2 , 10×2 , 12×2 , 14×2 , ... List some multiples of the first number

1×2 , 3×2 , 5×2 , 7×2

Multiples of 5: 2×5 , 10×5 , ...

1×5

List the multiples of the second number until there is a match

∴ The LCM for 2 and 5 is: **10**

(ii) 6 and 8

Multiples of 6: 6, 12, 18, **24**, 30, ... List some multiples of the first number

Multiples of 8: 8, 16, **24**, ...

List the multiples of the second number until there is a match

∴ The LCM for 6 and 8 is: **24**

We can use the list of prime factors for larger numbers to find the LCM by looking at the differences.

Find the LCM for these pairs of larger numbers

(i) 30 and 100

Prime factors of 30: 2, **3**, 5

List all the **prime** factors for both numbers

Prime factors of 100: 2, 2, 5, 5

Circle **all** the **different** factors in the smaller number

∴ The LCM for 30 and 100 is: $100 \times 3 = 300$

Multiply the larger number by the **different** factor

(ii) 24 and 388

Prime factors of 24: 2, 2, **2**, **3**

List all the **prime** factors for both numbers

Prime factors of 388: 2, 2, 97

Circle **all** the **different** factors in the smaller number

∴ The LCM for 15 and 388 is: $388 \times 2 \times 3 = 2328$

Multiply the larger number by the **different** factors

**Lowest common multiple (LCM)**

1 Find the lowest common multiple for these pairs of numbers.

a 3 and 9

b 5 and 10

c 4 and 6

d 5 and 6

e 6 and 7

f 12 and 16

2 Use the prime factors to find the LCM for these larger numbers.

a 60 and 108

b 42 and 150

c 168 and 180

d 210 and 385